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Identifying Mathematics Teachers’ Difficulties in Technology Integration in Terms of Technological Pedagogical Content Knowledge (TPCK)

Ayten Erduran, Burçin Ince

Abstract

A number of gaps were seen in the qualitative research on teachers using the TPCK model in our country, and this led us to conduct research focusing solely on Koehler and Mishra’s TPCK model (2005). To this end, the purpose of this study is to identify the difficulties high-school teachers face when integrating technology into their classes, and understand the reasons for these difficulties within the framework of TPCK. The method used was the case study. The participants were five mathematics teachers. The data collection tools were lecture plans, class monitoring, semi-structured interviews and a focus group interview. The data analysis techniques used were document analysis, descriptive analysis, content analysis and speech analysis. The findings showed that the teachers have difficulties with components of TPCK (PB-PAB-TPAB) and that the difficulties they face in integrating technology into the teaching process can be sorted under five main headings: (1) Teaching without planning, (2) Struggles to integrate different types of knowledge, (3) Lack of basic knowledge, (4) Errors in technological knowledge, (5) Lack of field-specific support. In order for the difficulties teachers face to be minimized, in-service training focusing on the use of technology needs to become more integrated, provide teachers with knowledge applicable to the classroom, and be given by experts in the field.

Introduction

“The only thing that does not change is change itself.”

Herakleitos of Ephesus

The use of computer technologies in Turkey began in 1960 in the Turkish General Directorate for Highways with the use of the IBM-650 Data Processing Machine and quickly began spreading among both state and private institutions. In 1984, through the acceptance of a commission established by the Ministry of National Education [MEB], the use of technology was introduced into educational institutions (Engin, Tösten & Kaya, 2010). Since the change in the teaching process in 2005 and the adoption of the constructivist approach, which gives greater importance to the student, in learning, technology has begun to enrich educational processes. In order for the quality of educational processes to increase the use of computer technologies needs to be combined with the constructivist approach, and this has become a topic of discussion. It has generally been thought that computer technologies will enrich the learning environment, help achieve meaningful learning and support the constructivist approach. Alkan (1991) discusses the importance of using educational technologies in order to provide individuals with a quality education, alongside the need to satisfying the different requirements and demands of the public, the need for the effective use of human resources and the need to achieve equal opportunity in education.

The Basic Education Project (Phase 1, 1998-2003; Phase 2, 2002-2007; MEB, 2003; 2007) and FATIH (Increasing Opportunities and Bettering Education Movement) project (2011–…) (MEB, 2011a) are two projects carried out as part of the effort to integrate the constructivist approach and computer technologies into education. They contributed to providing software and hardware support in schools, the setting up of computer labs and information technologies classes and providing each class with smartboards. Yet according to the 2007 data from the Directorate General of Educational Technologies of the Ministry of National Education, the computers provided to schools are mostly unused and going to waste. It was discovered that of 55224 schools only 3362 (6.08%) have been provided with equipment and software within the scope of the FATIH project since December 2010 and that although a budget of 4.3 billion TL was put aside for 2015 this percentage only went up to 10% (Dursun, Kirbaş & Yüksel, 2015). In schools provided with equipment and software, due to a
poor roll-out and lack of in-service training, the project could not be adequately implemented (Dursun, Kırbas & Yüksel, 2015) and the technologies required could not be used efficiently (Doğan, Çınar & Seferoğlu, 2016). Thus, the need for teachers who were able to effectively use technological equipment became a matter of importance. Koehler and Mishra (2009) also emphasized the teacher’s importance in the process of integrating technology and education.

But is it Enough for the Teachers to be able to Use Technological Devices?

Along with the importance of integrating computer technologies into the process of learning, a matter of key importance is the application of technology in the field of education. However good the technical equipment, the greatest responsibility for goals to be achieved lies with the teachers (Hollebrands & Lee, 2008). Knowing how technological tools such as computer software, scientific and graphic calculators are used means being able to effectively teach using these tools (Akkoç, Özmantar & Bingölbalı, 2008). Therefore, teachers must know about and understand the technology they use, and must be aware of why they have chosen these specific devices (Davies, 2011). Nevertheless, similarly to how the comprehensive knowledge regarding a subject does not guarantee being able to successfully teach it, being able to correctly use technology does not guarantee that it can be successfully integrated into education. The true problem is that teachers do not receive technological training that is combined with pedagogical knowledge and their respective fields (Powers & Blubaugh, 2005).

Since the expectations regarding what teachers are expected to know and be capable of as providers of education are ever-changing and dynamic (Association for Turkish Education [TED], 2009), there is a need to establish some basic proficiencies for the teaching profession. In 2008 and 2009 the Directorate General for Teacher Training and Education [ÖYEGM, 2006] specified and standardized proficiencies for teachers. The MEB-ÖYEGM 1st and 2nd term reports specify Content Knowledge, Content Education Knowledge, Attitudes and Values, Professional Development and Support of Mathematics Culture as the four proficiency fields for secondary education mathematics teachers, along with 14 sub-proficiency fields and 87 indicators of performance.

In 2000, the International Society for Technology in Education (ISTE) specified six technological standards and 23 indicators for teachers (NETS-T) under the heading “Technology Standards and Indicators”. In 2008, ISTE made revisions, specifying five technology standards and 20 indicators in total. In 2017, ISTE also published ISTE Standarts for Educators which includes two sub-titles – Empowered Professional and Learning Catalyst –, seven technology standarts and 25 indicators. In Turkey, Shulman’s (1986, 1987) cornerstone work, in which he categorized teacher knowledge into seven groups, has played a leading role. The groups of knowledge teachers must acquire are defined as:

- Content knowledge,
- Pedagogical content knowledge,
- Curriculum knowledge including class management and organization,
- Knowledge of students and their characteristics,
- Knowledge of the learning environment and its requirements (knowledge of context),
- Knowledge of functions, objectives and values regarding education and the philosophical and historical bases underneath them (Shulman,1987).

Shulman’s model (1986, 1987) which labels teacher proficiency “Pedagogical Content Knowledge” has been developed further by a number of researchers (Pierson, 1999; Koehler & Mishra, 2005). The model developed by Koehler and Mishra (2005) was named the “Technological Pedagogical Content Knowledge Model” (TPCK-TPACK).

Technological Pedagogical Content Knowledge Model

The Technological Pedagogical Content Knowledge Model developed by Koehler and Mishra (2005) and seen in Figure 1 consists of three types of information: Technological Knowledge, Content Knowledge and Pedagogical Knowledge.
Figure 1. Technological Pedagogical Content Knowledge Model (Koehler & Mishra, 2005).

Pierson (1999), defined technology integration in his doctoral thesis as the combining of technological, pedagogical and content knowledge or the integration of technology in pedagogy and content for the first time. The inception of the idea of TPCK is Mishra’s (1998) theory of three types of knowledge, which are identified as content, theory (defined as the opposite of pedagogy) and technological knowledge (Schmidt, Baran, Thompson, Mishra, Koehler & Shin, 2009). TPCK is a model that explains the relationship between and complexities in these three main areas of knowledge (Koehler & Mishra, 2008; Mishra & Koehler, 2006). The model is the intuitive interpretation of the three types of knowledge along with the suitable pedagogical methods and technologies (Schmidt & et al., 2009). As seen in Figure 1, it consists of seven components: Technological Knowledge (TK), Content Knowledge (CK), Pedagogical Knowledge (PK), Pedagogical Content Knowledge (PCK), Technological Content Knowledge (TCK), Technological Pedagogical Knowledge (TPK) and Technological Pedagogical Content Knowledge (TPCK).

### Table 1. Definitions of the components of the TPCK model*

<table>
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<tr>
<th>Component</th>
<th>Definition</th>
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<tr>
<td>Pedagogical Knowledge (PK):</td>
<td>Teaching methodologies and processes; contains classroom management, evaluation, developing teaching plans and student learning (Schmidt, Baran, Thompson, Mishra, Koehler &amp; Shin, 2009).</td>
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<tr>
<td>Content Knowledge (CK):</td>
<td>Teachers’ knowledge of the specific field that will be taught or learned (Koehler &amp; Mishra, 2009).</td>
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<tr>
<td>Technological Content (TK):</td>
<td>Teachers’ knowledge of digital calculation devices and software (Tabach, 2011).</td>
</tr>
<tr>
<td>Pedagogical Content Knowledge (PCK):</td>
<td>Understanding a specific subject within the discipline and how to use certain teaching practices in order to help students in the learning process (Cox &amp; Graham, 2009).</td>
</tr>
<tr>
<td>Technological Pedagogical Knowledge (TPK):</td>
<td>Motivating students to achieve cooperative learning through the use of technology (Cox &amp; Graham, 2009).</td>
</tr>
<tr>
<td>Technological Content Knowledge (TCK):</td>
<td>The knowledge of how to suitably present specific subjects within the given content using existing technologies (Cox &amp; Graham, 2009).</td>
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<tr>
<td>Technological Pedagogical Content Knowledge (TPCK):</td>
<td>Teachers’ ability to coordinate subject content knowledge with specific forms of presentation using new technologies in order for students to learn (Guerrero, 2010).</td>
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</table>

* The definitions chosen are those with the most explanatory value and which are most suitable for practice, as judged by the authors.

It is not enough for a teacher to have the three types of knowledge that provide the basis for this model separately. Rather, the sub-components TCK, PCK, TPK and the intersection of all three, TPCK must all be found within a teacher. The model emphasizes the critical role of teachers within the classroom rather than the importance of the components and suggests that teachers must bring them together using technology in the most suitable way (Bull, Park, Searson, Thompson, Mishra, Koehler & Knezek, 2007). What is expected from teachers is that they will possess all three types of knowledge and be capable of integrating them effectively. According to Koehler & Mishra (2009), the expertise of a teacher who can transition between technological, pedagogical and content knowledge is very different and more extensive than of a teacher who is successful in each separately.
Schmidt et al. (2009) have emphasized that prospective teachers must be observed in the classroom and many studies have been conducted to that end (Akköç, Özmantar, Bingölbaş, Demir, Baştürk, & Yavuz, 2008; Bilici, 2012; Niess, 2005; Hardy, 2010; Aygei & Voogt, 2012; Angeli & Valanides, 2009; Angeli & Valanides, 2013). Most of the research regards prospective teachers and the status of active teachers as issues of interest. Additionally, while there are many quantitative studies in Turkey (Bozkurt & Çiçek, 2011; Pamuk, Ülken & Dilek, 2012; Archambault & Barnett, 2010; Schmidt et al., 2009), the limited qualitative research available (Akköç et al., 2008; Bilici, 2012, Akkaya, 2009; Ugurlu, 2009; Pamuk, 2012; Tokmak Sancar, Yelken Yanpar & Konokman Yavuz, 2013; Tokmak Sancar, 2015) shows that the TPCK model, has been studied constructed on the PCK model and examined in a number of different contexts such as multiple representations, student difficulties and errors, teaching methods and strategies, assessment and evaluation, application of the curriculum and content knowledge.

There are currently projects backed by TUBITAK, titled “Investigation of Prospective Mathematics Teachers’ Knowledge of Assessment in the Context of Summative and Formative Assessment” and “Educational Practices to Provide Science and Technology Teachers with Technological Pedagogical Content Knowledge in Turkey.” Worldwide, there are studies conducted by the Milken Family Foundation (MFF), the International Society for Technology in Education (ISTE), the National Institute of Education (NIE) as well as the Teaching Teachers for the Future (TTF) Project, supported by the Australian Government’s Department of Education, Employment and Workplace Relations (DEEWR).

On becoming aware of the shortcomings of the qualitative research regarding TPCK model in Turkey, we wanted to conduct a study focusing on the TPCK model and its components. In their research, Baran and Bilici (2015) examined a total of 30 studies on TPCK in Turkey between January 2005 and December 2013, and found that 22 of the studies were quantitative while seven were qualitative. They found that five of the studies were conducted with working teachers while 24 were conducted with trainees. This study is thus important in filling a gap in the literature and for understanding the current situation regarding mathematics’ teachers and the underlying causes of the challenges they are faced with within the framework of TPCK.

The research objective is to identify the challenges teachers face in integrating technology into mathematics education and revealing the underlying reasons for these difficulties within the framework of TPCK. After the findings are evaluated the intention is to propose various recommendations. In light of the objective, the main research question was defined as: “What are the difficulties mathematics teachers face in integrating technology into the teaching process in terms of Technological Pedagogical Content Knowledge (TPCK), and what are the reasons for these difficulties?” The related questions are a) “The lack of which component(s) of TPCK (TK, PK, CK, TPK, TCK, PCK, TCPK) is the reason mathematics teachers have difficulties in integrating technology into the teaching process?” b) “What seem to be the underlying causes of mathematics teachers’ difficulties in integrating technology into the teaching process?”

Method

The Research Model

The research holds true to the constructivist approach which allows individuals to be observed in their natural environments and for individual meanings to be examined and interpreted. In light of our approach, the qualitative method, which does not allow for natural and physical phenomenon regarding humans to be separated as dependent or independent, and expects the researchers to engage in an in-depth examination of humans in their own environments perceptively rather than objectively, was used (Yıldırım & Şimşek, 2011).

In the research the difficulties faced by teachers during technology-aided mathematics lessons, and the reasons for these difficulties were assessed within the framework of TPCK. With the research objective in mind the research model used was that of the case study, as it is the qualitative research method that is most interested in generalizations. A case study is defined as a research design that allows the researcher to analyze a situation, often a program, event, action, process or one or more individuals in-depth (Creswell, 2016); a method that examines in-depth one or more events, environments, programs, social groups or other related systems (McMillan, 2000). Because case studies examine daily phenomenon in their actual context (Yin, 1994, pp.13), it is thought that using a case study in this research will contribute to the evaluation of TPCK components in the teachers’ natural environments.
Participants

The participants consisted of teachers working in Anatolian and equivalent schools affiliated with the Ministry of National Education in the city of Izmir. To determine which schools would be selected criterion sampling was used. The criterion here was that the schools chosen had technological equipment in the classrooms. The choice of teachers was made using maximum variety sampling. To this end, the sample consisted of teachers who claimed to be proficient, moderately proficient and unproficient in technology-aided education. From among teachers with smartboards in their classrooms, two were sampled from each proficiency level, resulting in a total of six participants. Because one of the teachers later withdrew from the study, the final sample consisted of five participants. All of the participants were given pseudonyms. All of the teachers in the sample had graduated from Educational Science programs and had attended a number of in-service training courses on technology. Additionally, all of the teachers had either graduated from or were still attending Masters’ programs.

<table>
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<th>Table 2. Features of the participants.</th>
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<tr>
<td><strong>Emre</strong> was teaching in an Anatolian high school in the Izmir region and was in the 15th year of his service. He had completed an MA, was open to developing his skills and had an interest in and inclination for technology.</td>
</tr>
<tr>
<td><strong>Gül</strong> was teaching at a multi-program Anatolian high school in the Izmir region and had been a teacher for two years. She was continuing her MA and was a young teacher who had a close relationship with technology and believed it should be used throughout the day.</td>
</tr>
<tr>
<td><strong>Özge</strong> taught at an Anatolian high school in the Izmir region and had been a teacher for 17 years. She told us that she had completed her MA, tried to improve her skills, and had attended Geogebra courses within the scope of the Fatih Project.</td>
</tr>
<tr>
<td><strong>Nesrin</strong> was a teacher at a multi-program Anatolian high school in Izmir and was in her 3rd year of service. She was continuing her MA, was willing to develop her technological skills and had attended a number of courses to this end.</td>
</tr>
<tr>
<td><strong>Halide</strong> was teaching at an Anatolian high school and was in her 25th year as a teacher. She had completed her MA, was not very interested in technology in her professional or daily life, but was open to improving this.</td>
</tr>
</tbody>
</table>

The Role of the Researchers

The researchers collected the data in its natural environment, and analyzed it in the back garden. The researchers were experienced in observation and interviewing and had done work on technology-aided mathematics education. Throughout the process the researchers conducted non-participant observation. They took care not to affect the participants with uncalled-for comments, not to take an active part in the observation and not to shape the study during the interpretation process.

Process

Written consent was acquired from the participants at the beginning of the research process. Before the teaching process, the teachers were asked for their lesson plans. The research began by monitoring lessons with the aid of an observation form prepared by the researchers; this was followed by semi-structured interviews. The research then concluded with a focus group interview. The research data was gathered during the 2013-2014 academic year and the approval of the Ethics Committee of the Izmir Provincial Directorate for National Education was received. As the research aims were to determine how the teachers integrated technology into their lessons, the lessons monitored were limited to four hours of teaching of the first two gains of second degree functions and their graphics for 10th grade mathematics in order to avoid other factors affecting the process. To keep the diversification of gains from pulling focus away from the main research objective the in-class observation was limited to four hours. The gains focused on are: “Can explain and draw the corresponding graphic for a second degree function with one unknown” and “Can solve problems that can be modeled with second degree functions and their graphics” (MEB, 2011, pp.26). The additional statement “Information and communication technologies can be used” suggests that the use of technology was suitable for the subject taught.

In order not to disrupt or affect the class observations were made from the back row of the classroom during the normally scheduled teaching hours. After the observation process, a semi-structured interview was conducted with each participant. The interviews took place at an agreed date and time, in a quiet environment where the participant would feel comfortable. The content of these interviews included the observed lessons. After the
observation and interviewing processes were completed for all participants, a convenient date, time and place was established for the focus group interview.

Data Gathering

The teachers were asked to provide four hour-long learning plans for the first two gains of second degree functions and their graphs sub-learning field for 10th grade mathematics. The reason for this request was to determine the teachers’ perceptions regarding integrating technology in lessons and their skill in integrating the three types of knowledge (pedagogical, technological and content). Along with the observation form, developed by the researchers, four hours of class monitoring were conducted in order to see how these learning plans were applied in the classroom. The observation form was prepared based on the Association of Mathematics Teacher Educators [AMTE] (2006) and ISTE/NETS-T (2000, 2008) standards and, for the content of the TPCK model, the literature review. The observation form was validated by three academics in the Mathematics Education department with regards to the observability of the standards specified on the form and their suitability to the established components. In addition, three teachers’ classes were monitored for an hour beforehand, in order for necessary revisions to the form to be made.

In order to understand the difficulties teachers had when integrating technology into mathematics lessons and the causes of these difficulties in detail, one-on-one interviews were conducted with each teacher. The interviews were semi-structured, and a question sheet was prepared following a literature review. A draft interview form was presented to three faculty members and three teachers, and given its final form based on the revisions suggested. In addition, a focus group interview was conducted with the teachers in order to acquire multiple data. The focus group meeting was conducted using a focus group sheet consisting of flexible questions, and both audio and video-recorded. Two members of faculty and two teachers were consulted for revisions. In order to increase the reliability of the study, multiple methods were used for information-gathering.

Data Analysis Techniques

The lesson plan data was examined using document analysis, which is the systematic examination of existing records and documents (Best, 1959; as cited in Karasar, 2012; p. 183). When the lesson plan data were acquired, they were analyzed with regard to structure (where it was written, how it was written), content (which gains were put first, whether it was prepared based on books or the curriculum), and objectives (whether it was for technology-aided learning or preparation for central examinations). The class monitoring data acquired through the observation form was examined using descriptive analysis, because this allows for data to be summarized and interpreted based on predetermined themes (Özdemir, 2010); the themes in this case being the components of TPCK (TK, PK, CK, TPK, TCK, PCK, TPCK). One of the researchers monitored the classes using the observation form, making notes and taking photographs when necessary with the permission of the participants. Qualitative reliability indicates the consistency of the researcher’s approach when examined by other researchers (Gibbs, 2007; through Creswell, 2012). In order to establish this, two researchers conducted the analyses for each teacher collaboratively. In this way consistency between coders was achieved simultaneously.

The data acquired through the one-on-one semi-structured interviews with the participants was analyzed through content analysis, which consists of four steps: (1) coding the data, (2) finding the themes, (3) organizing the codes and themes, (4) defining and interpreting the findings (Yıldırım & Şimşek, 2011). The interviews were transcribed and analyzed through the stages of coding, theme-finding and organization. The data acquired from the focus group meetings were analyzed using speech analysis, which focuses on describing what people say, and systematically examining their gestures, mimics and body language during speech (Ekiz, 2009). The interview was transcribed and themes were generated based on the reasons for the teachers’ difficulties with the TPCK components.

Strategies for Questioning the Validity of Findings

The validity of the findings was ensured by collecting data from multiple sources, data diversity and excessive data-gathering. After the gathering and analysis of data, the participants were contacted again for member checking. Additional time was spent with the participants in their teaching environments, outside the four hours assigned for classroom observations.
Results and Discussion

Findings and Analysis of the Difficulties and the Reasons for the Difficulties Teachers Face with the Components of TPCK

In order to identify the difficulties teachers face when trying to integrate technology into lessons within the framework of TPCK lesson plans were requested from the participating teachers. Two of the teachers, Özge and Halide, prepared presentations instead of written plans before class, and submitted these presentations as learning plans. When the presentations were examined it was seen that they consisted of the lecture chapters of exam preparation textbooks. Nesrin and Gül kept written plans to hand, and referred to those during class. It was seen that the teachers’ class preparation mostly consisted of the subject headings and example problems to be solved in class, with no planning for getting the attention of or motivating the students. Emre, on the other hand prepared presentations on the whiteboard and taught the lesson according to the examples and headings there. It was seen that none of the teachers conducted a written evaluation of the students in the end of class, but Nesrin included assessment and evaluation questions during the lesson in order to attract the attention of the students. All of the teachers included the methods and techniques they would be using. When viewed in this regard, it was seen that none of the teachers included PK in the learning plan. As an extension of this, there were also no findings for TPK, PCK and TPCK. It was seen that the teachers only gave a number of examples with different qualities in terms of CK.

Teachers Considering Themselves Proficient in Technology: Emre and Gül.

When Emre’s PK was examined, the interviews showed that while he knew the discovery learning method and the constructivist approach to learning he had difficulties putting this knowledge into practice. For example, he had prepared a PowerPoint presentation as a motivational activity: when presented with a photograph of the Malabadi Bridge, with the caption “What is this?” the students simply replied “The Malabadi Bridge” and the teacher moved on to the next slide. Similarly, when presented with a photograph showing the distance a basketball travels when it bounces, the class showed no reaction. In these cases, the teacher is expected to get the students to focus on the subject at hand, but the teacher here was unable to redirect the students through the constructive or any other approach. It was also observed that the teacher ensured class management through eye contact with the students. He did not make any effort to assess and evaluate them in accordance with the study plan. It was considered that this teacher had difficulties with his PK. In accordance with these findings, he mentioned that PK was taught in university but remained theoretical. He also stated that he did not find himself proficient in terms of PK, but that through his experiences as a teacher he had found his own style of teaching. In the interview, Emre said that he did not have much knowledge of newly developing educational methods, techniques or strategies. Since he did not hear much about them, he did not feel curiosity or a need to follow these developments.

When his CK was scrutinized, it was seen that he gave the correct definitions, was capable of relating concepts to each other (relation-function-equation, parabola-in equation-notation table), addressed the critical parts of concepts (min-max values, symmetry axis, peak point and the relation between them) and used mathematical language and symbols correctly (\( f(\alpha)_{\text{max}} \)). Interviews also showed that he followed developments in the field, had a subscription to a mathematics journal and rated his content knowledge as 4 out of 5. The different types of examples he used in class also help illustrate how he was a teacher proficient in CK.

![Figure 2. TK example for Emre](image)

When the teacher’s TK was scrutinized, it was seen that while he sometimes had minor problems turning the smartboard on and off, he was generally confident and capable of using the toolbars in the software effectively. For example, when Emre wanted to collectively erase what he had written from the board, he circled the area he wanted gone, clicked on the “erase” toolbar and marked an “X” on the area in the circle (seen in Figure 2). His
in-class conduct supported his claim in interviews that he easily adapted to new technology. The teacher had strong TK.

When Emre’s PCK was examined, it was seen that he used examples from daily life, such as referring to the parabola catenary as being “like a bowl” and likening the parabola figure to the concave mirror used in physics classes, thus showing he was capable of making connections between different disciplines and using multiple representations. In the one-on-one interview, he stated that he tried to use the discovery learning strategy and often used the question-and-answer technique to achieve this. Below is an in-class dialogue to illustrate this:

Teacher: It is asking you for the minimum value of $x^2 - 2x$. What would you do to find it? How would you use your knowledge? Raise your hand and wait. (He goes to the students side.)
Student 1: I put it in x parentheses. If we equate it to 0…
Teacher: Well why did you equate it to 0? It’s not an equation; it’s an $f(x)$ function. It doesn’t say it equals zero.
Student 1: But it’s its smallest value.
Teacher: Do we always have to equate it to 0 for its minimum value?
Student 1: I don’t know, but here I equated it to 0 to find it.
Teacher: Is it because we always said 0 for the smallest value?
Student 2: I also put it in x parentheses. I assigned 1, inside the parentheses became (-1). (-1) is small. When I assigned different numbers it was either greater than 0 or equal to 0.
Teacher: So you used trial and error?
Teacher: OK. Yes? (He gives another student permission to talk)
Student 3: What if we assign a range of values?
Teacher: We know the ranges of values, functions more or less from last year. If you give it a range of values, or a domain you can indeed find the minimum and maximum values for that set, but the $f(x)$ function is a polynomial and it is defined in all real numbers from negative infinity to positive infinity. There is no need to restrict the range of values.
Student 4: I found 0 or 2 in x’s place, it’s one of the two.
Teacher: You set up an equation; in the equation you gave x 0. Or rather one of its roots is 0 the other is 2. Did you try 0 or 2, what was the minimum value for each?
Student 4: It was 0.
Teacher: So how do you know it’s the smallest? Are you sure it is the smallest?
Student 5: Teacher, let’s first assign 0 as the smallest value. In case there are values smaller than 0, I made it smaller than 0. I sent x off to the other side, it became x minus x is smaller than minus x. That didn’t do it.
Teacher: So?
Student 5: -1. Sorry, 1.
Teacher: You gave x the minimum value 1? What became the minimum value then?
Student 5: -1
Teacher: So you just tried two values?
Student 5: No teacher. How do I explain? (Silence)
Teacher: So the smallest value is -1, how do you decide it can’t be any smaller? Are you sure?

It was seen that the teacher wanted to apply discovery learning and guided the students with questions, but due to the class’s readiness and the teacher not managing to ask suitable questions the technique could not be completed. In this case the students could not be brought to the intended point. There were issues with the teacher’s PCK. In addition to the observational findings, the teacher said that the teaching technique and strategy he most often used was “discovery learning”, and continued, “I mean I try to get them to associate it with things. Like equations, second degree equations. What does that count as? Probably the discovery approach.” Although this indicated a lack in the teacher’s PK, it was observed that the teacher’s lessons were well constructed and that they were conducted in a way that kept the students motivated. Thus, it was thought that he was able to make up for difficulties in his PK through his CK.

When Emre’s TPK was examined, it was seen that the work he did on the interactive board using colored pens increased the students’ interest in the class and motivated them. That he went to the classroom to turn on the board during breaktime, and that after taking the class register and signing the class register sheet he plugged his flash drive into the board and drew the curtains so the board would not be reflective showed that he made the classroom suitable for using technology. However, he was not able to support his presentation with suitable questions during class. In his learning plan, it was only written that the presentations he had prepared in Starboard would be used. The teacher’s TPK needed developing. Although during the interview, the teacher
stated that using technologies that corresponded to the students’ interests and skills would get the students’ attention and increase student learning. When questioned on how much he used technology for classroom management, his reply was “I don’t. Can’t say I do, I’ve never tried.” These answers indicated that the teacher could not put into practice the things he knew theoretically. During the focus group interview, Emre said, “Frankly we have problems with integrating pedagogy and technology. We feel ambivalent about what to choose. I personally do. I find it difficult deciding which technology to use within the subject.” This shows that the teacher found it difficult to choose which materials to use technology with (TPK). He emphasized that he had problems integrating his PK and TK.

When the teacher’s TCK was examined, it was discovered that he knew programs such as Geogebra, Cabri 3D, Derive, Sketchpad and Graph Analysis; that he used Cabri and Derive and intended to use Graph Analysis and Geogebra for future subjects. During the class monitoring it was seen that he used the interactive board, utilized presentations as a motivational tool with regard to the shape of parabolas, but did not use any mathematics software, animations, videos or e-books. In particular, during interactive in-class activities he was only seen to use the interactive board software, Starboard. The teacher’s TCK required improvement. During the interviews, he pointed out that he knew software such as Geogebra, Cabri and Graphic Analysis yet did not use them during class or for following the lessons. He added that he had recently begun to use these technologies in other lessons, and that he was unable to follow new developments in the field.

![Figure 3. TPCK example for Emre](image)

When the teacher’s TPCK was examined, it was seen that he could relate the \( f(x) = x^2 \) graph, and second degree functions in general, to tables and algebraic statements using Starboard and could use technology-aided multiple representations about the subject (Figure 3). In addition, in his final class, Emre gave function graphs and their corresponding algebraic statements and asked the students to pair them together to enforce their understanding of the leading coefficient relation to the graph of second degree functions.

Thus the teacher was able to prepare material suited to the subject (the presentation serving as a motivational exercise), prepare material using technology that made learning easier for his students (the graphs and statements pairing exercise for illustrating the relation of lead coefficient and graph) and put together presentations using the interactive board software. He mostly preferred to use the materials he had developed specifically for the subject, along with the direct instruction method. It was observed that he had difficulties in applying the materials he had prepared with the Starboard software in the classroom. It was seen that the teacher had difficulties with TPCK. In his interview he stated that he was unable to integrate technologically-aided materials and teaching methods and techniques, that working on it himself was not enough, and that he felt a need for additional training. These all indicate that issues with PK and TCK were preventing the teacher from integrating the two. It was thought that the teacher had difficulties with integration (TK-PK, TK-CK and PK-TCK) and required technological support specific to the content.

When Gül’s PK was examined it was seen that, partly due to her being a recent graduate, while she had theoretical knowledge she found it very difficult to translate this into practice and was not even capable of using the question-and-answer technique. It was also observed that she did not follow the learning plan, and could only manage to successfully give some of the expected gains, failing at others. It was noted with interest that, despite this, in interviews she said she preferred the discovery learning strategy and adopted the constructivist approach. During the four hours of class time it was seen that Gül adopted the expository teaching strategy and the lecture method. Below is a section of her lesson.

Teacher: What was the symmetry axis? The line \( x=r \). How did we find \( r \)? With \( \frac{b}{2a} \). What is a here? It’s \( x^2 \)’s coefficient. 1. \( b \) is \( x \)’s coefficient which is \(-2\). \( c \), the constant is \( 5 \). Minus \(-2\) divided by two times \( 1 \). The result is \( 1 \). So \( x=1 \) is found as the symmetry line. Let’s find \( k \). What’s the formula for \( k \)?
As can be seen from this extract from Gül’s lesson, she delivered information without allowing the students to think, answered her own questions and made the students write down the information. In this regard, it was seen that she had many deficiencies in PK.

When her CK was scrutinized, it was seen that she presented the peak point concept in function graphs, directly with their formulas. She did not mention the critical points of this concept – minimum and maximum values, converting to a perfect square – or the relation between the function and its graph. In the interviews, she rated her content knowledge as 3 out of 5. From the observations and interviews, it can be inferred that Gül had problems with CK. As a reason for the difficulty she had with CK, she explained that although she had an inquiring mind she did not do much research in her field. It also seemed that she did not always put her learning into practice and was unable to explain to students how the expressions were formed.

When Gül’s TK was examined, it was seen that she could use the toolbars on the interactive board (she could change the color and thickness of the pen), and did not encounter any difficulties using the board. In one class only, Gül, while using the Geogebra software, was unable to write the functions in the entry point due to the cursor being in another space. She noticed this when no changes happened to the board when she expected the graph to appear. After gazing at the screen for a few seconds to figure out the problem she brought the cursor to the entry point and rewrote the function using the screen keyboard. But because she wrote \( f(x) = 2x + 3 )2 - 8 \) instead of \( f(x) = 2(x + 3)^2 - 8 \) warnings such as “change name, enter new name for function g” appeared on screen. Her response was to say “Oh well” and continue with the lesson without spending more time on the matter. This episode lasted for approximately three minutes. This shows that the teacher was able to use the technologies she used often in class appropriately but had issues solving problems when she encountered difficulties. In the one-on-one interviews she pointed out that she easily adapted to a new technology, could use programs such as basic Microsoft Office and Video Converter, and that she spent 4-5 hours on the web each day. It appeared that the teacher’s TK was substantial.

When Gül’s PCK was examined, it was seen that at the beginning of the first class she showed videos from the Education Informatics Network (EIN), as a motivational activity. It was observed that she used examples from daily life and used multiple representations, using expressions such as “the route the dolphin takes in the video” or “the water creates a curve when it squirts from the fountain” to explain parabola curves. In the interview, she stated that she could make connections with other disciplines such as physics and chemistry. After the teacher showed the students the peak point and solved examples, she wrote on the board that “The legs of the graph for the function \( f(x) = ax^2 + bx + c \) would be turned upward if \( a>0 \) and the function’s minimum value would be at \( f(r) \); and if \( a<0 \) the graph’s legs were turned downward the function’s maximum value would be at \( f(r) \)” without asking the students any questions to obtain their opinions or include them in the lesson. The fact that the teacher never spoke about the concepts of minimum and maximum value, and asked and answered the question herself is telling. It was clear that she was lacking in PCK. This lack can be explained as a result of her inability to put her theoretical PK into practice during class. Although she stated in her interview that when students lacked motivation she would put forward a problem and create a discussion, and that she made them learn through doing, she was not seen to do this during her classes. After monitoring the lessons, it was seen that due to her lack of CK she found it difficult to achieve student learning. The questions asked by the students indicated that the concepts of equations and second-degree functions were being understood as separate from each other. The reason for the teacher’s difficulties with PCK can be attributed to neither her PK nor CK being sufficient on their own.

When Gül’s TPK was observed, it was seen that she could not manage the class with the aid of technology. In particular, when she used Geogebra there were times she lost control of the students, in comparison to when she used the whiteboard. This was again due to the hitches she encountered using Geogebra. Yet the fact that she used the Geogebra software to illustrate that the \( f(x) = ax^2 + bx + c \) function could be also written as \( f(x) = a(x - r)^2 + k \), through an example did catch the interest of the students. From the reactions of the students, it was seen that she was able to make the learning process easier for students through use of technology. It was observed that in this regard the teacher had difficulties with TPK. During her interview, she said that she was unable to gain control over the classroom after tablet computers were passed out to the students. She said that instead of taking an interest in the lesson, they started playing games and this undermined classroom management. The situation indicates that the teacher’s issues with PK also affected her TPK.
When Gül’s TCK was observed, it was seen that in one of her classes she used GeoGebra software for a short period (Figure 4) and that she showed a video on the topic. In the interview she said that she made very limited use of Cabri 2D; that she used e-books, followed EIN, and followed technological developments in her field through journals. Yet there were no findings indicating she was sufficient in TCK within her classroom performance. Thus, since the teacher’s TK was sufficient and her CK required development, the difficulties with TCK can be attributed to her lack of CK and inability to integrate the two knowledge areas.

When her TPCK was examined, it was seen that there was a limited use of multiple representations with the aid of GeoGebra (only algebraic notation and graphs). However, her lack of both CK and PK was demonstrated during these activities, because in class she only attempted to use the technology. While focusing on drawing the graph she strayed further from explaining what she was attempting to explain. She could not effectively manage the process by integrating technology and direct instruction. It was seen through her use of video that she could use technology-aided teaching materials, but was not able to integrate them with PK. Other than these, it was observed that she did not prepare or use technology-aided teaching materials. She stated that preparing materials is a difficult process and that she preferred to use readily available material instead. It was observed that the teacher had difficulties with the TPCK. She explained the reasons for this difficulty as being that preparing material is a challenging process, that she thought she would have problems in the classroom when she could not adapt to the material during the lesson, and that the issues she encountered could be caused by herself, the material, the students or the environment. Although the teacher had sufficient TK, the problems she had with PK and PCK naturally affected her TPCK, and caused her difficulties in integrating her TK and PCK. Gül was seen to lack in basic knowledge (PK and CK) and have difficulty with integrating knowledge (TK-CK, TK-PCK).

**Teachers Who Consider Themselves Neither Proficient Nor Unproficient in Technology: Özge and Nesrin.**

When Özge’s PK was examined it was seen that her method and technique knowledge was weak, both in theory and practice; that she only knew the question-and-answer technique and the lecture method and used these in class. In addition, although she stated that she put the students and their needs at the center of her teaching, she did not pose questions that put the students at the center and made them think, and usually resorted to the expository teaching strategy. For example, it was noticed that the teacher lectured with her back toward the class, facing the board for approximately 1.5 minutes.

Teacher: We represent the coordinates of the peak point with r and k. The apsis is r, the ordinate is k. When we draw a perpendicular line from the apsis, the graph will always be separated into two equal parts. For example, these two distances (she shows the distances to y of two points on the same ordinate) are equal? Equal to each other. Why? Because these have to be symmetrical according to the x=r line. I’m taking two different points in the same way. When I look at the line that I’m drawing, what do these two distances have to be? Equal. In short whether the legs look up or down when I draw the x=r line what will happen to the parabola? It will be split into two halves. I will have a symmetry axis. OK? Is there anything you want to ask about up to this point? (Silence) Let’s continue.

It was seen that Özge did not have issues with classroom management, the students listened quietly and took notes; she had good communication with her students and joked with them during class. It was found that Özge had struggles with PK, both theoretically and practically. As reason for her issues, she explained that the teaching methods and techniques had changed from the ones she had learned at university and that she could not follow the new methods. It was also observed that although she followed the curriculum the teacher’s theoretical knowledge was not sufficient, and although she found herself proficient pedagogically, she was unable to put her knowledge into practice.
When her CK was evaluated, it was seen that she could not relate concepts to each other, only relating functions and parabolas and she did not touch upon the critical point of converting the parabola into a full square. It was seen that she had no trouble using mathematical notations. In her one-on-one interview, when asked how she rated her content knowledge between 1 and 5 she gave herself 4.5 points, which shows that she is confident in her abilities. Yet classroom observations did not support her evaluation of herself. For example, though expected to relate the peak point of second-degree functions to second degree equations, she expressed \( r = -\frac{b}{2a} \) as \(-\frac{b}{a}\) half the sum of roots. The teacher’s CK required improvement.

When her TK was examined, it was seen that she had no hesitation in using the interactive boards and its software, and comfortably used the back step, carry to drawing board, change pen color and thickness toolbars. During the presentation she had prepared for her second class, when asking the students an exercise question, it was seen that she did not carry the exercise onto the drawing board as a page, instead cutting the question out and carrying it using the toolbars. When on the drawing board, she carried the question to the top of the page and magnified it for the students to see it more clearly. In addition, when the teacher erased a solution she had written on the drawing board before realizing the students had not yet been able to copy it down, she was able to bring the page back after looking at the toolbars for about ten seconds. In her one-on-one interview, she said that she did not regularly use the internet, and that she used it for an hour at most on weekends. She also stated that she could easily adapt to new technology, and when she encountered problems, she found solutions by trying again. The teacher was seen to have TK. In addition, she pointed out during the focus group interview that not all content is suitable for the use of technology, and that she mostly preferred to use technology for subjects that had a visual component, or that were difficult in terms of student learning. It was concluded that she saw technology as a tool to be used only when needed.

When her PCK was examined, it was seen that she did not use any motivational activities, did not walk around the classroom much, only interacted with a few students in the front rows, and could not use questions that led the students to think. In addition, she did not ask questions that understood what students were thinking, and did not lead the class in that direction. In the one-on-one interview she stated that she had sometimes tried to use the discovery learning strategy at the beginning of classes, but no examples of this were found during the in-class observations. Also, while not observed in class, she said that she made connections with other disciplines (such as referencing the biology class when explaining binomial expansions). It is thought that the teacher had difficulties with PCK. Her struggles with PK and her lack of CK were the reasons for her deficiency in PCK.

When her TPK was examined, it was seen that using the cordless mouse she could easily access the board even from the back of the class, she drew the curtains and made the learning environment suitable for technology use. She also pointed out that she could access the board, even from the back of the class and that she could manage the class through the use of technology. Although she had no issues with TK, her difficulties with PK and inability to integrate the two types of knowledge led to the conclusion that the teacher’s TPK required improvement. The teacher expressed this difficulty with integration, saying, “I have difficulties integrating pedagogy and technology” during the focus group.

![Figure 5. Example of Özge’s TCK](image)

When the teacher’s TCK was examined, it was seen that in the last minutes of her final class she used the animation she had learned in the Geogebra course (Figure 5), but not in any other of her classes. In the last minutes of the last class, she showed an animation she had prepared during the course, showing the changes to \( a \) on a parabola belonging to the function \( f(x) = ax^2 + c \).

Özge was not sure how to activate the sliding bar in the material and the researcher had to inform her that “you need to click ‘play’ on the right tab, teacher.” This showed that the teacher had not fully grasped the toolbars. For this reason, she was also unable to increase the intervals on the coordinate system and the parabola moved in a very restricted space. This led to comments from the students:
Student 1: Teacher, we can’t understand anything.
Student 2: Teacher, what are they doing there by themselves?

It was observed that the students were critical and that the changes in parabolas according to the a value were not really understood. In the interview she mentioned hearing of software like MathType and Cabri 3D, but said that she did not use them and that she liked the web page Matemaic.as. The difficulties she had using the Geogebra software for drawing parabolas also showed that she had failings in TCK. It was thought that her TCK needed improvement. The teacher pointed out that she visited websites about her field and that she tried to follow technological developments. She also said that she tried to implement the learning from a Geogebra course she attended on the last day of class monitoring, but found that she required more practice.

When her TPCK was examined, it was seen that she drew graphs using the interactive board, was able to use multiple representations and could prepare mathematical materials (presentations) using technology. In the PowerPoint presentation she prepared, it was seen that she solved the exercises with her students in class and then when she moved on to the next slide it gave the solution to the same exercise, copied and pasted from the textbook. Because this slide was not really used and the students were not guided with effective questions, it was redundant, and negatively affected the practicality of the material. Özge was observed to have difficulties with TPCK. In her interview, she discussed these difficulties, saying “I think, ‘How can I impart, deliver a subject to a student?’ I struggle with this.” It was thought that the teacher’s difficulties with PK also affected her TPCK, and that, in addition, she was unable to integrate her PK and TCK. Özge was seen to be lacking in basic knowledge (PK and CK) and to have difficulties with integration (TK-PK, PK-TCK) although it was observed that she did not have errors in TK (means-end).

When Nesrin’s PB was examined, it was seen that she showed videos as motivational activities, frequently used the discovery learning strategy and the question-and-answer technique, supported constructive learning and could manage the class. In her interview, she emphasized that for students of all levels the most effective method is discovery learning, which was enough for her to get students to think; that she tried to get students to work out specific information through discussion and that sometimes she used the drama technique in class. In the interview, she also said that she got the students’ attention by preparing beforehand and coming to class with a clear plan and schedule; she started with examples from daily life to pique the students’ interest and did not have issues with class management. In the interview, it was learnt that she used observation forms, self-evaluation forms, short exams at the end of lessons and questions asked at the beginnings and ends of lessons in order to assess and evaluate. For example, at the end of the last lesson, she had the students write down two questions as homework. It was seen that she followed the new lesson plan, with some small changes. For example, it was seen that she did not cover the first gain of the sub-learning field second degree functions, which was “Can you calculate the minimum and maximum values of the statement $f(x) = ax^2 + bx + c$”. It was also observed that she discussed the minimum-maximum value critical point when examining the peak point of the parabola. It was seen that the teacher’s theoretical and practical knowledge overlapped, and that she did not have issues with PK.

When her CK was examined, it was seen that she had trouble connecting concepts to each other. For example, when defining second-degree functions, she framed the expression $f(x) = ax^2 + bx + c$, and wrote the second-degree equation next to it. Moreover, as she wrote she said “This is our second degree equation and what we call a parabola is the graph of this second degree equation” showing that she was not able to understand the difference between a function and an equation. In addition, she did not mention operations such as converting to full square, or finding the minimum and maximum values, which are critical points. In the interview, Nesrin was asked to evaluate her content knowledge between 1 and 5 and she gave herself a 3-4. Because she was unable to form connections between concepts and did not touch upon some points critical for gains to be fulfilled it was observed that she had difficulties regarding CK.

When her TK was examined, it was seen that she used the toolbars (colored pens, new page, take back etc.) in the interactive board software, and that when she wanted to erase what she had written she drew an x on it and used the “erase” toolbar. It was also observed that when she turned off the page on the drawing board before realizing the students had not yet been able to copy it down, she attempted to bring the page back but failed. Nesrin was seen to need improvement regarding her TK. During her interview, she said that because she had not had any training she found using the interactive blackboard challenging at first but then improved through practice. She emphasized that she thought she needed to improve her TK, saying, “I think as women we are less proficient in this regard. I’m not precious about it.”
When Nesrin’s PCK was examined, it was seen that she used the drawing program on the interactive board and never used the whiteboard. To link the shape of the parabola to daily life she used the expression “curves like bowls”; she conducted classes with the students at the center and attempted to get students to construct knowledge. While this was not observed, in interviews she stated that she connected her teachings to physics and biology. It was seen that the teacher favored the constructivist approach to learning and student-centered education. For example, she drew the \( f(x) = ax^2 \) parabola, defining the coefficient as the integer chosen by the students and then asked the students to examine and comment on the graph.

Teacher: What happens to the function graph as the a coefficient increases?
Students: It gets narrower.
Teacher: So the legs come closer to y. On the contrary, when the a coefficient decreases what happens?
Students: The legs get wider.

Afterwards the teacher wrote “in \( f(x) = ax^2 \) according to the value given to a, the legs of the parabola move closer to or further from y” to summarize the exercise on the smartboard. At the end of the final lesson, Nesrin wrote two questions linking parabolas to daily life, stating, “These are our questions on the subject in daily life.” She conducted assessment and evaluation at the end of the lesson by asking the students to take out papers and giving a small written exam consisting of 5-6 questions. In her interview, she said that sometimes she had the students evaluate the papers and sometimes evaluated them herself. The teacher was found to have PCK.

When her TPK was examined, it was observed that she showed videos and used colored pencils to gain the attention of her students. Also it was seen that she used the interactive board throughout the lessons monitored. The teacher’s tendency to write the exercise down on the interactive board led to the students talking amongst themselves, negatively affecting classroom management. She pointed out in her interview that when she failed to use the board effectively, the class tended to get distracted. She added that when she first began using the interactive board she had difficulties with class management. She also said in her interview that she used technology to improve students’ learning by drawing graphs and asking for common points, showing videos and giving examples drawn from daily life. This shows a consistency between the information she gave in interviews and the in-class observations. Nesrin often used the Geogebra software in class. For example, she had the students find the differences in the legs as the a coefficient in the function \( f(x) = ax^2 + bx + c \) took on the values -2 and 3. It was observed that Nesrin’s TPK needed developing. In her interview, she said that at the beginning of her career she had had difficulty with classroom management due to her not having a command of technology and the interactive blackboard, and it was observed during her classes that she was unable to develop solutions to the problems she faced when using the technology. During the focus group the teacher stated that although she used technological tools in order to improve student learning, she did so in an unbalanced way and that trying to get the students’ attention through technology caused a digression from the subject, indicating that the teacher was unable to integrate her existing knowledge areas (PK-TK).

![Image](image-url)

**Figure 6. Example of Nesrin’s TCK**

When the teacher’s TCK was examined, she pointed out that she had learned to use mathematics software such as Geogebra, Sketchpad, Drive, Cabri3D during her undergraduate education and that she used Geogebra and Cabri 3D in her classes. It was observed during lessons that she favored the Geogebra software (Figure 6) with which she could make drawings with ease and which contains animations, and that she played a video about second-degree functions on the EIN database.

Differently to the other teachers, Nesrin did not use ready-made materials from Geogebra. The fact that she used graphics that she drew while teaching, with values she asked the students for, showed that she in fact had TK and was capable of using it for the subject of second-degree functions. It was concluded that even with some lack of TK, she managed to overcome this and positively affect her TCK. In the interview she mentioned that she followed the Vitamin Teacher website. She also stated that she did not have a great understanding of the
logic behind mathematical software, and for that reason, she was not able to prepare tasks that were good enough; and although she used the Cabri software, she was not proficient in it. The teacher was observed to have difficulty integrating her TK and CK.

When Nesrin’s TPK was examined, it was observed that she often drew parabolas on the Geogebra software and the drawing board, and used mathematics education materials like showing the video on second-degree functions from the EIN database. Nesrin was observed to conduct student-centered lessons, and to attempt to construct student knowledge. For example, for the graph of the \( f(x) = ax^2 + c \) function, she made changes to the \( c \) value using first positive integers and then negative integers given by the students, and drew the graph using Geogebra. The dialogue below followed:

Teacher: Let’s see what happened, class, what can we say about this?
Student: It moved downward. Teacher, now at the beginning you gave the graph for 3x, when two units come forward, teacher, it goes upward, or in the positive direction. Teacher, if we give the negative direction, teacher, it will move downward. For example, it can be thought like, legs downward.
Teacher: And what axis does this movement happen on, class?
Students: Y. The y axis.

The teacher thus generated the critical points of the subject with her students and got them to make inferences, and wrote notes on the interactive board afterwards. Similarly, in the final lesson she drew the graph of the \( f(x) = a(x - r)^2 \) function with the \( a \) and \( r \) values given by the students on Geogebra. She drew the students’ attention to the changes on the graph according to the changes in the \( a \) and \( r \) values. Finally, she was seen to draw the \( y = 2x^2 \) parabola, followed by the \( y = 2(x - 2)^2 \) parabola using Geogebra and wrote notes on how the changes happened using the interactive board. The teacher was observed to employ the constructivist learning approach when teaching second degree functions, yet not to treat the subject in the order given in the curriculum. It was thought that the teacher’s TPK required improvement. In her interview, she stated that it was difficult to prepare material that would make student learning easier. It is also thought that she was not very proficient in using technology and found it difficult to integrate her TK and PCK. Nesrin was seen to have issues with integration (TK-CK, PK-TK, TK-PCK), and there were errors (wasting time) in her technological knowledge.

**The Teacher Who Considers Herself Unproficient: Halide.**

When Halide’s PK was examined it was seen that she preferred the expository method of teaching, and even though she attempted to use the discovery learning strategy she ended up presenting a definition herself. For example for \( r \), which is one of the coordinates of the peaking point, she gave the formula \( r = \frac{b}{2a} \), and asked the students where this equation came from. The following dialogue occurred:

Teacher: Why is \( r = \frac{-b}{2a} \)?
Student: From the equation?
Teacher: We could follow the equation. What else?
Student 2: \( x_1 \) plus \( x_2 \)?
Teacher: We can also go through the intersection points. Very well seen.
Student 3: Teacher, from the sum total of the roots?
Teacher: Why do we divide by 2? Is there such a connection?
Student: The added total of the roots is \( \frac{-b}{a} \).
Teacher: The total of the roots is \( x_1 + x_2 = \frac{-b}{a} \). Our friend says it must have to do with that.
Student: Oh it’s half that.
Teacher: What does half mean?
Teacher: Isn’t the symmetrical halved? The distances should be equal. This point (showing \((r,0)\)) should be the midpoint of these two. How do we find the middle? We add and divide by two.

As can be gathered from the dialogue, Halide tended to favor more traditional teaching methods in her lessons. Although she was to touch upon the concepts of peak point and axis of symmetry, the fact that she left these to the end of the topic suggested that she made changes to the order of gains in the curriculum. She also expressed in her interview that she found the lecture plan too complicated and got confused as to how much she should
teach. It was observed that the teacher achieved classroom management through eye contact and by calling upon the students. The teacher was observed to have difficulties with PK. In the interview she also confessed that her theoretical knowledge was insufficient when asked about teaching techniques and strategies, saying, “There’s induction, deduction, questioning, explanation teaching techniques; there’s learning through doing, cutting and pasting.” However, in practice she was seen to adopt the constructivist approach and attempt to get the students thinking. During the focus group, she stated that she thought her struggles were due to the pedagogy education she received in university not being adequate, and her own inability to put it to practice. She thought that rather than trying to improve her PK through books, taking courses would be more helpful.

When her CK was examined, it was seen that she could make connections between concepts (function-equation). When going through the items required for a parabola to be drawn, the item “Find the points where the parabola intersects with the axes” was examined. During the examination, the teacher gave the value of 0 and found the equation “\( x^2 + 3x - 4 = 0 \)” and said, “This is familiar to us. It’s an equation of the second degree with one unknown. We’ve learnt everything about this. We know whether or not it has roots. We know if there is one or two, we even learned how to examine the notations without finding the roots just last week.” These expressions show that she is capable of relating equations and functions, and the intersection points on the graph to the roots of the equations. In the interviews, Halide also said that she did not follow the developments in her field or mathematics books very much; that she would rate herself as a four when asked to rate her content knowledge between 1 and 5, and that while she was confident she could not always answer every question off the top of her head. In this regard, it can be said that Halide’s CK was sufficient.

When Halide’s TK was examined, it was seen that she prepared PowerPoint presentations for class; was not able to use the toolbars of the interactive board and only touched the board to change pages. It was also observed that Halide always came and turned the board on at break time and got it prepared for class beforehand. When at the beginning of the second lesson while she was going through the pages, when she changed a page by mistake it was seen that she hesitated and made no attempt to bring the page back. It was observed that she preferred the whiteboard to the interactive one when lecturing. She pointed out that she did not use technology very much in classes, other than to prepare presentations and project questions onto the board. In her interview, she said that she could learn new technologies but did not trust herself much in that regard. She emphasized that because she could not use the technology well in the classroom, she thought it would end up wasting time. It was thought that Halide had most difficulties with TK. As reason for her difficulties, she said that she first became familiar with computers in the year 2000, during her Master’s, and that she mostly used the computer for research and reading newspapers. She said that she did not want to waste time trying to use technology in class, and take away from the students’ learning time; and that she intended to learn how to use it in her free time before using technology in class.

When Halide’s PCK was examined, it could be said that she tried to relate subjects to daily life. For example, she showed that the parabolas legs would change in accordance with the coefficient of \( x^2 \) and formed sentences as such: “\( x^2 \) has increased and become \( 2x^2 \) but the legs won’t open up, they’ll get closed. Therefore, if it was \( \frac{1}{2} \) \( x^2 \) then it would be more open. Like a flower, it opens and closes. As it becomes greater it closes, as it gets smaller it opens”; “I’m going to draw a shape like a glass” (Figure 7). It was observed that she planned her classes well and stuck to her plans. In the interview she said that she uses concept maps, finds causal relations, and makes the students create proofs to see certain things for themselves. During the observations she tried to find the coordinates of the peak point. But because of her lack of PK she directed the class without taking the students’ opinions into account. There were issues in her PCK. These issues were thought to stem from her lack of PK, although her use of reflective learning in order to better herself in her 25 years of teaching had helped to make up for some of the shortcomings with the PK. Also, her PK was stronger in practice than in theory, and the fact that she was able to integrate it with her CK reduced the issues she had with her PCK.
When her TPK was examined it was found that she tried to get the class’s attention with the presentations, but because she only used the interactive board as a projector, and had difficulty coping with problems during its use, there were decreases in the students’ motivation. Halide stated that she wanted to use technology in class, but because she was incapable of using it effectively and proficiently she was worried she would lose control of the classroom and therefore preferred not to. The teacher had difficulties with her TPK. This difficulty was thought to be due to her shortcomings in TK. In her interview, the teacher stated that technology negatively affected classroom management, and that technology could not give students efficient feedback and revision in terms of assessment or evaluation. The teacher’s belief that technology made no contribution to education kept her from using her existing TK and from developing it further. The teacher’s difficulties with PK were also a factor here, and the struggles with both types of knowledge kept them from being integrated.

When Halide’s TCK was examined, it was seen that she had heard of Geogebra and Cabri, had attended a course to learn Geogebra and knew how to use PowerPoint. She mainly used the interactive board as a projection and questioning device, and did not use any of the software specific to the field or any animations, videos or other technology-aided visual resources. In the interview, she said that she tried to visit mathematics forums occasionally, and that she wanted to follow developments in the field of mathematics, was enthusiastic but was unable to use technology. The teacher was seen to struggle with TCK. In her interview, she pointed out that she was not very interested in mathematics forums, and did not visit these sites. This indicated that while she would have liked to follow developments in her field, she did not believe herself capable of using technology. The reason for this was thought to be that although the teacher believed in the necessity of using technology in education, she was not able to internalize this belief. The teacher’s shortcomings in TK kept her from integrating it with her CK.

When her TPCK was examined, it was found that she could prepare material using technology (PowerPoint presentations, pairing the leading coefficient of the function with its graph), but was not very proficient in it. For example, she designated the coefficient of the function \( f(x) = ax^2 \) as 1 and -1, drawing the graphs separately on the whiteboard. She then continued the lesson on the whiteboard, drawing the functions for the coefficient values for a as 2 in the first graph and -3 in the second and then made comparisons along with her students. Later she turned to the presentation she had prepared on the interactive board and skipped the pages which showed the instructions “Draw the graph for \( f(x) = 2x^2 \)” and “Draw the graph for \( f(x) = -3x^2 \)” consecutively. This led to the thought that the teacher did not follow the instructions and had not mastered the material. The teacher struggled with her TCPK. She pointed out that she found it difficult to prepare technology-aided material. She stated that using Geogebra supports the students’ learning of the concept of periods as it provides visualization, but also emphasized that for this to be true the teacher needs to be proficient in the technological tool. The real reason for the teacher’s difficulty was thought to be her lack of TK, which was the hidden part of the iceberg. Halide was seen to have no difficulties with integration; on the contrary, she was able to integrate her PK and CK. She encountered difficulties due to there not being sufficient support specific to the field, and the main source of her problem came from lacking basic knowledge (TK).

**Conclusion**

We examined the ways in which deficiencies in components of TPCK negatively affected the participants’ process of integrating technology into their teaching. It was found that certain participants struggled especially with PK (Emre, Gül, Özge and Halide), PCK (Emre, Gül and Özge) and TPCK (Emre, Gül, Özge and Halide). The reasons for the participants’ struggles in integrating technology into their teaching process have been grouped into five categories. These categories are teaching without a plan, struggles with integration of types of knowledge, lack of basic knowledge, errors in technological knowledge, and lack of field-specific support.

**Teaching without a Plan**

The participants in the research were asked for lesson plans of the lessons to be observed, yet only Gül and Nesrin prepared written lecture plans, and those were copied from preparation guidebooks for central examinations. It was seen that the three other teachers prepared presentations, either with PowerPoint or on the drawing board. As Polly (2010) suggests, preparing lesson plans, and receiving feedback and revisions to these plans, make the process of integrating technology into lessons easier for teachers. This lack of planning is likely to be one of the causes for the participants’ difficulties in using technology to aid them in the classroom.
Struggles with Integration of Types of Knowledge

TK and PK: Emre and Nesrin were seen to have difficulties integrating TK and PK, although they were proficient in both types of knowledge separately. Niess (2005) emphasized that trainee teachers tend to remain unproductive in terms of teaching strategies (PK) to be used with technology (TK). Özge verbally expressed difficulty with integrating her PK and TK. All five of the teachers were observed to struggle with integrating TK and PK, and this is thought to come from them struggling with classroom management, assessment and evaluation, and how to gain students’ attention. The teachers’ negative beliefs, such as “I’ve never tried it with classroom management” or “It negatively affects classroom management” also inhibited integration.

TK and CK: Emre, Gül and Nesrin were seen to have difficulty integrating their TCK. Although he possessed the two knowledge areas separately, Emre, similarly to Polly’s (2010) findings, was seen to be interested only in gains (CK), or only in technological materials (TK), at any one time, in effect neglecting TCK. TK and PCK or PK and TCK: Emre and Özge were observed to be unable to integrate their TK and their TCK. It was predicted that this would be caused by the teachers being lacking in PK and being unable to combine PK and TCK. Hoffer and Swan (2006) pointed out that teachers, even with a substantial accumulation of CK, PK and TK, might have issues with their knowledge when it came to academically constructing the lesson.

Gül was observed to struggle with PCK and not be able to integrate TK and PCK, and Nesrin was observed only to be unable to integrate her TK and PCK. Bos (2011) also observed this shortcoming, emphasizing the importance of teachers using technology (TK) that is mathematically and pedagogically (PCK) focused in order to establish conceptual learning. The reason for Nesrin’s need to improve her TPCK was her issues with the process of developing material, the teacher’s lack of proficiency in technology and her inability to integrate her TK and PCK. This is supported by Bilici’s (2012) findings that teachers lack knowledge on how to prepare teaching materials with the aid of technology during the integration process.

Lack of Basic Knowledge

Of the participant teachers, all except for Nesrin were seen to be lacking in PK, and require improvement in both theoretical and practical knowledge. Gül and Nesrin were found to need improvement in CK. Other than Halide, who found herself lacking in terms of technological knowledge, all participants were found to have sufficient TK. In this case, all of the participants were found to have shortcomings in at least one of the three basic knowledge areas that makeup TPCK. It is thought that these deficiencies in the basic types of knowledge lie at the heart of the struggle teachers have with TPCK. Pamuk, Ulken and Dilek (2012) also found that teachers find themselves to be unproficient in using technology, and that the underlying reason was that they had received an insufficient education in basic content, technology and pedagogy knowledge during their training.

Özge said that she was aware of her difficulty with PK and that she had tried to improve herself and overcome her shortcomings throughout the years. Schmidt et al. (2009) found that the number of years teachers had spent practicing their profession improved their scores in the questionnaire they used in their research. In accordance with this, it was also observed that Özge, as an experienced teacher, had been able to overcome her shortcomings to a significant degree. Özge and Gül were found to have good TK, while their PK remained mostly theoretical, and were found to have issues with all types of knowledge requiring PK (PCK, TPK and TPCK). Polly (2010) similarly found that teachers who taught mathematical knowledge efficiently and used technology to increase students’ mathematical abilities tended to have sufficient TCK, but tended to struggle with PK and knowledge areas that include PK, such as PCK, TPK and TPCK.

Although Halide had attempted to improve herself in terms of technology, she was found to have difficulties with TK, due to her self-doubt and lack of enthusiasm. As Archambault and Barnett (2010) point out, TCK, TPK and TPCK are all strongly linked to each other and exist together in a helical structure; thus, the teacher also had shortcomings in TCK, TPK, and TPCK. Bozkurt and Cilavdaroğlu (2011) also found that teachers’ negative attitude and lack of self-confidence with technology caused their issues with TK to have a domino effect on all other components of knowledge. In addition, Lehtinen, Nieminen and Viiri (2016) emphasized that in order to encourage teachers to use technology throughout their professional lives, their beliefs regarding technology must be improved throughout the whole of their bachelor’s education.

As Cheng and Zhan (2012) point out, teachers are required to first improve themselves in the basic types of knowledge (such as TK, PK and PCK), in order to then improve the components that contain these types of knowledge (TPK, TCK and TPCK).
Errors in Technological Knowledge

As Bilici (2012) suggests, Halide’s use of the interactive blackboard only to show PowerPoint presentations suggested that she used technology as an end rather than a means. Özge, on the other hand, was found not to struggle with TK, but stated that not all subjects are suitable to technology use, and that she preferred to use technology with subjects that had a visual component or that were challenging in terms of student learning when integrating technology into the teaching process. This indicates that she saw technology as a means to be used when needed, rather than an end. The findings of Akkaya’s (2009) study with trainee teachers suggest that trainees should try to integrate technology into their lessons only when necessary, seeing it as a tool rather than an objective, that the subject should not take a back seat to technology, and that using technology for every subject is meaningless. These findings support the examples of Özge and Halide.

Nesrin was not seen to have any difficulties with her PK, and during her interview she stated that while the students’ level was high in terms of technology, TK was not her strong point and that she found it difficult to gain the students’ attention and achieve classroom management (TPK) with technology. This indicates that the teacher was of the opinion that technology did not support student learning and that it created problems with classroom management. Bilici (2012) supports this, arguing that achieving classroom management is especially difficult in a classroom where the students are of a high socio-economic status and have access to technology both inside and outside of school.

Lack of Field-specific Support

In their research, Doering, Veletsianos, Scharber & Miller (2009) found that teachers need support that is not just limited to additional courses but that continues after they return to the classrooms. The clearest indicator of this is Halide’s explanation during the focus group interview that after receiving an in-service training course on Geogebra, she used it in class but that she could not take it any further than that. Similarly, Özge set aside the last five minutes of class time to practice parabola drawing and adding sliding bar operations, as she had learned in an in-service training course, but was seen to encounter problems. This shows that the teachers needed additional training and support upon returning to the classrooms after attending courses.

During the focus group interview, the participants stated that they had difficulty figuring out how to adapt educational technologies into their own lessons, and that training courses were designed for teachers of all fields, making it difficult to work out how to adapt technology to their own field. Guerrero (2010) points out that each field has its own educational objectives, and that because every field will have different types of technology and practices, the TPCK model must be specific to each field and training courses must be structured according to this. Also on this subject, the teachers argued that the training provided was mostly basic and theoretical, when receiving an education on how to use this information in the classroom would have been more beneficial to them. Voogt, Fisser, Pareja Roblin, Tondeur and van Braak (2013) share this opinion, and emphasize the importance of trainee teachers gaining experience with lessons enriched with technology rather than giving technology-based lectures.

Recommendations

The study examines the difficulties middle school mathematics teachers face when integrating technology into their teaching process, and the underlying causes for these difficulties. The conclusion reached is that these difficulties are caused by shortcomings in the PK, PCK, and TPCK components of the TPCK model. It is also strongly emphasized that the participants’ being lacking in TK significantly affected all other components. In addition, when the reasons for these difficulties were examined, it was seen that five main issues emerged. These were: teaching without a plan, struggles with the integration of knowledge areas, shortcomings in basic knowledge, fallacies in technological knowledge and lack of field-specific support.

With regard to the teachers’ deficiencies in knowledge, there need to be activities that will make them aware of what they lack and they must be given help to overcome these shortcomings. In addition, providing teachers with PK, CK and TK by themselves is not enough. As Pamuk, Ülken and Dilek (2012) also point out, in addition to gaining knowledge in the basic fields, teachers also need to gain the skills to use this knowledge by relating them with other knowledge areas as well. As Koehler and Mishra (2005) suggest, in order to go beyond the traditional technological education, the teacher must have experience of technology that increases
the strong ties between content, pedagogy and technology knowledge. Receiving an education that encompasses these three types of knowledge from an expert in their own field, supported with examples, will likely encourage teachers to improve. An education of this nature should ideally not be limited to one or two weeks, but allow for the teachers’ improvement to be monitored fully.

Awareness-raising activities are necessary in order to overcome the idea that teachers can integrate technology into the classroom simply by preparing PowerPoint presentations, or projecting questions onto the board. To prevent anxiety about using technology, teachers need to be given the option of sustained in-service training courses, and be encouraged to attend such courses. Teachers need to be specifically encouraged to prepare technological materials and be provided with support in order to prepare material that matches the approach required by the curriculum.

Technological tools should have a place at every stage of university education and academic staff should set an example for the students. Trainee teachers need to be given comprehensive and applied pedagogical and content education both during their undergraduate and postgraduate education. Studies need to be conducted with teachers who have not received or are not receiving a master’s education in order to establish how they integrate technology into their teaching processes.

References


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**Author Information**

Ayten Erduran  
Dokuz Eylül University  
Buca School of Education  
Izmir, Turkey  
Contact e-mail: erduranayten@gmail.com

Burçin Ince  
Dokuz Eylül University  
Buca School of Education  
Izmir, Turkey